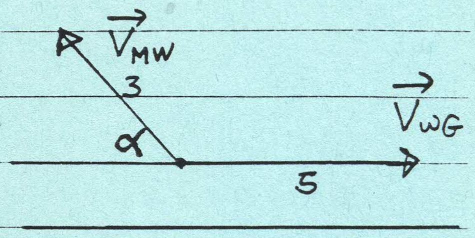
1975 Q2 H.L.

2. A man wishes to swim across a river 60 m wide. The river flows with a velocity of 5 m/s parallel to the straight banks. and the man swims at a speed of 3 m/s relative to the water. If he heads at an angle α to the upstream direction, and his actual velocity is at an angle θ to the downstream direction, show that $\tan\theta = \dfrac{3\sin\alpha}{5 - 3\cos\alpha}$.

Prove that $\tan\theta$ has a maximum value when $\cos\alpha = \frac{3}{5}$. Deduce that the time taken for the man to cross by the shortest path is 25 s.

$V_{MW} < V_{WG}$



$$\vec{V}_{MW} = -3\cos\alpha\,\vec{i} + 3\sin\alpha\,\vec{j}$$
$$\vec{V}_{WG} = 5\vec{i}$$

$$\vec{V}_{MG} = \vec{V}_{MW} + \vec{V}_{WG}$$
$$= (5 - 3\cos\alpha)\,\vec{i} + 3\sin\alpha\,\vec{j}$$

$$\Rightarrow \underline{\underline{\tan\theta = \frac{3\sin\alpha}{5-3\cos\alpha}}} \qquad \left(= \frac{V_y}{V_x}\right)$$

$$\frac{d}{d\alpha}(\tan\theta) = \frac{(5-3\cos\alpha)3\cos\alpha - 3\sin\alpha\,(+3\sin\alpha)}{(5-3\cos\alpha)^2}$$

$$= \frac{5.3\cos\alpha - 3^2}{(5-3\cos\alpha)^2}$$

$$\underline{\underline{\frac{d}{d\alpha}(\tan\theta) = 0}} \Rightarrow \underline{\underline{\cos\alpha = \frac{3}{5}}}$$

$$\Rightarrow \vec{V}_{MG} = \left(5 - \frac{9}{5}\right)\vec{i} + 3\left(\frac{4}{5}\right)\vec{j}$$

$$= \frac{16}{5}\vec{i} + \frac{12}{5}\vec{j}$$

$$\underline{\underline{\text{Time to cross} = \frac{60}{(12/5)} = 25 \text{ secs}}}$$